Teaching Primary Mathematics for Mastery

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Introduction
Learning from The Chinese Paradox

The paradox does not exist because good quality teaching actually takes place in Chinese mathematics Classrooms (Huang 2004)

What does it mean to master something?

- I know how to do it
- It becomes automatic and I don’t need to think about it- for example driving a car
- I’m really good at doing it – painting a room, or a picture
- I can show someone else how to do it.
Mastery of Mathematics is more.....

- Achievable for all
- **Deep** and sustainable learning
- The ability to build on something that has already been sufficiently mastered
- The ability to reason about a concept and make connections
- Conceptual and procedural fluency

Teaching for Mastery

- The belief that all pupils can achieve
- Keeping the class working together so that all can access and master mathematics
- Development of **deep** mathematical understanding
- Development of both factual/procedural and conceptual fluency
- Longer time on key topics, providing time to go deeper and embed learning
What is Depth?

Partitioning and Combining
Partitioning and Combining

Part whole relationships

7 is the whole
3 is a part and 4 is a part
Representing the Part - Part Whole Model
Attention to Structure
Teaching Primary Mathematics for Mastery

Depth shown in children’s work
8 flowers and 2 flowers

5 apples and 2 apples
Amy

6 + □ + 4 = 13

\[
\begin{array}{ccc}
 & & 13 \\
\hline
6 & | & 4
\end{array}
\]

6 + [3] + 4 = 13

23.11.15
WALT add three 1-digit numbers

5 + 4 + 9 = 18
6 + 2 + 4 = 12
6 + 3 + 2 = 11
3 + 2 + 5 = 10
Using the bar model
Developing Depth/Simplicity/Clarity

Equal and not equal
Attention to structure

And not a hungry crocodile in sight!
Ralph posts 40 letters, some of which are first class, and some are second.

He posts four times as many second class letters as first.

How many of each class of letter does he post?

<table>
<thead>
<tr>
<th>Class</th>
<th>Letters</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st</td>
<td>8</td>
</tr>
<tr>
<td>2nd</td>
<td>32</td>
</tr>
</tbody>
</table>

40 letters
He posts four times as many second class letters as first.
How many of each class of letter does he post?

40 ÷ 5 = 8
8 x 4 = 32
1st Class 8 letters
2nd Class 32 letters
Ralph posts 40 letters, some of which are first class, and some are second.

He posts four times as many second class letters as first.

How many of each class of letter does he post?
Meeting the needs of all pupils - The road to differentiation

Inclusion is important, but maybe we need to think about it in a different way.

Teaching Primary Mathematics for Mastery

Examples from Chinese teachers’ slides and Chinese textbooks
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Examples from Chinese teachers’ slides and Chinese textbooks

Review2:

How to illustrate these fractions on the diagram.

\[
\begin{align*}
\frac{2}{3} & \quad \frac{1}{4} & \quad \frac{4}{9} & \quad \frac{2}{3}
\end{align*}
\]
**Thinking 1:**

Garfield likes to eat cake. Today, he ate $\frac{2}{8}$ of the cake. Not satisfied, he ate $\frac{3}{8}$ of the cake later. How much cake did Garfield eat in total?

$$\frac{3}{8} + \frac{2}{8} = \frac{5}{8}$$

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**Formulation:**

$$\frac{3}{8} + \frac{2}{8} = \frac{2 + 3}{8} = \frac{5}{8}$$
**Rules of fraction addition**

\[
\frac{3}{8} + \frac{2}{8} = \frac{3+2}{8} = \frac{5}{8}
\]

**Keep the denominator the same and add the numerators.**

Chinese teachers do not see repetition and understanding as separate but rather as interlocking processes, complementary to each other (Waktins & Biggs, 2001).

**Providing Challenge**

Shanghai Textbook Grade 3
Use fractions to express the coloured parts.

Conceptual Variation
**True or False?**

\[
\frac{3}{8} + \frac{2}{8} = \frac{5}{16}
\]

\[
\frac{3}{9} - \frac{2}{9} = \frac{1}{9}
\]

\[
\frac{2}{14} - \frac{1}{7} = \frac{1}{7}
\]

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**Looking at all aspects of the concept**

Tasks which challenge and provoke reasoning

2 paper tapes were broken, can you guess which original paper tape is longer?

**Why? How do you get your answer?**

Variation to develop depth
Think: Which line is longer?

First: \[ \frac{1}{2} \]

Second: \[ \frac{1}{3} \]
Consider how variation can both narrow and broaden the focus
Taken from Mike Askew, Transforming Primary Mathematics, Chapter 6

Variation Theory in Practice

Compare the two sets of calculations
What’s the same, what’s different?

Set A                      Set B
120 – 90                   120 – 90
235 – 180                  122 – 92
502 – 367                  119 – 89
122 – 92                   235 – 180
119 – 89                   237 – 182
237 – 182                  502 – 367

Variation leads to
Intelligent Practice

Shanghai Practice Book
Intelligent Practice

In designing [these] exercises, the teacher is advised to avoid mechanical repetition and to create an appropriate path for practising the thinking process with increasing creativity.

Gu, 1991
Variation offers a systematic way to look at mathematical exercises in terms of what is available for the learner to notice. (Marton, Runesson & Tsui, 2003)

Let’s teach for:

Deep
And sustainable learning for all pupils